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# **Freshman Physics Laboratory**

## **Instrumentation: The CRT Oscilloscope**

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# Appendix A

## The Cathode Ray Tube Oscilloscope

### A.1 The Cathode Ray Tube Oscilloscope

The *cathode ray tube oscilloscope* is essentially an analog<sup>1</sup> instrument that is able to measure time varying electric signals. It is made of the following functional parts (see figure A.1):

- the cathode ray tube (CRT),
- the trigger,
- the horizontal input,
- the vertical input,
- time base generator.

Let's study in more detail each component of the oscilloscope.

#### A.1.1 The Cathode Ray Tube

The CRT is a vacuum envelope hosting a device called *electron gun*, capable of producing an electron beam, whose transverse position can be modulated by two electric signals (see figures A.1 and A.7).

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<sup>1</sup>Hybrid instruments combining the characteristics of digital and analog oscilloscopes, with a CRT, are also commercially available.

When the electron gun cathode is heated by a wire resistance, because of the Joule effect it emits electrons . The increasing voltage differences between a set of shaped anodes and the cathode accelerates electrons to a terminal velocity  $v_0$  creating the so called electron beam.

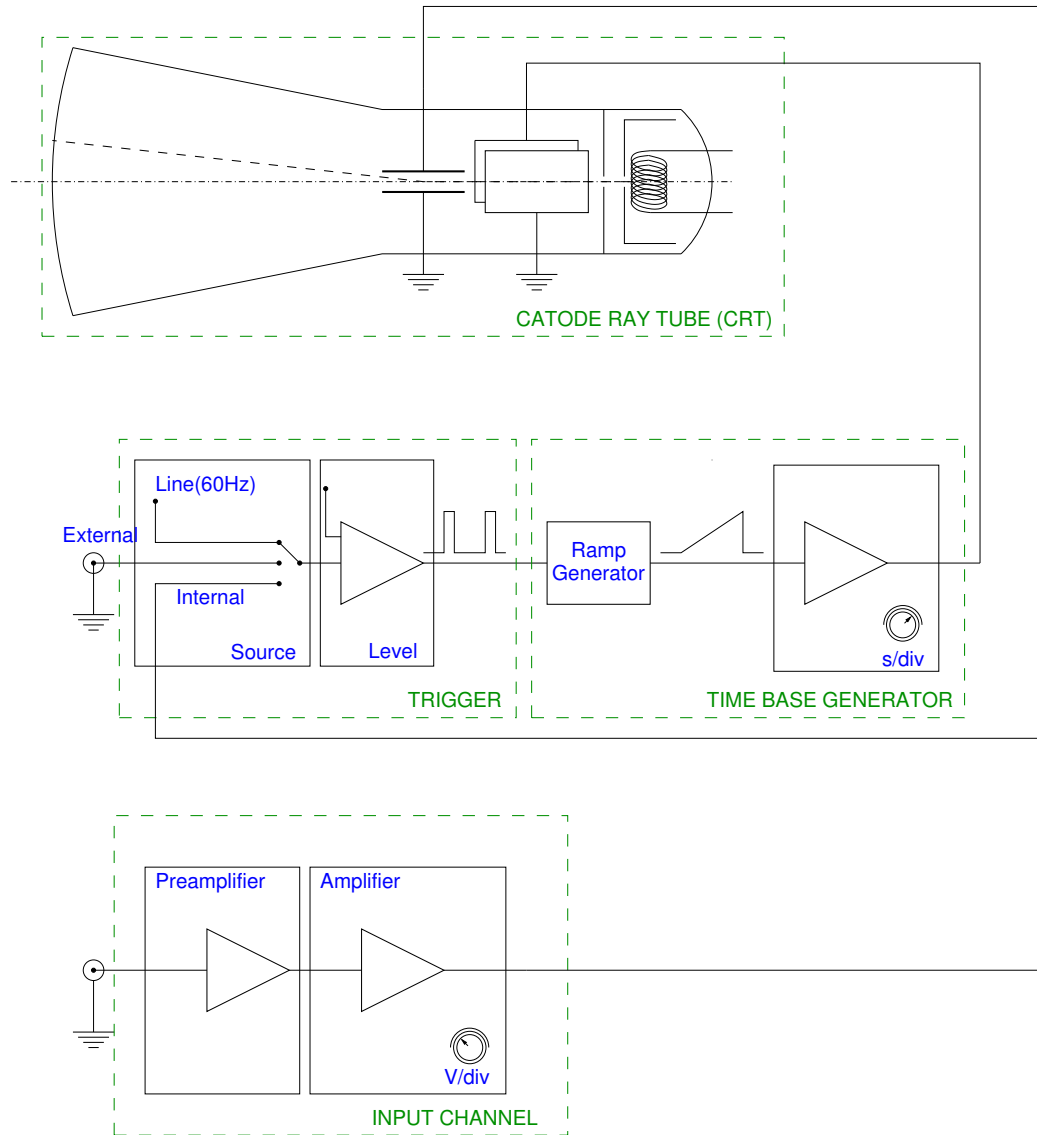


Figure A.1: Oscilloscope functional schematics

The beam then goes through two orthogonally mounted pairs of metallic plates. Applying a voltage difference to those plates  $V_x$  and  $V_y$ , the beam is deflected along two orthogonal directions ( $x$  and  $y$ ) perpendicular to its direction  $z$ . The deflected electrons will hit a plane screen perpendicular to the beam and coated with florescent layer. The electrons interaction with this layer generates photons, making the beam position visible on the screen.

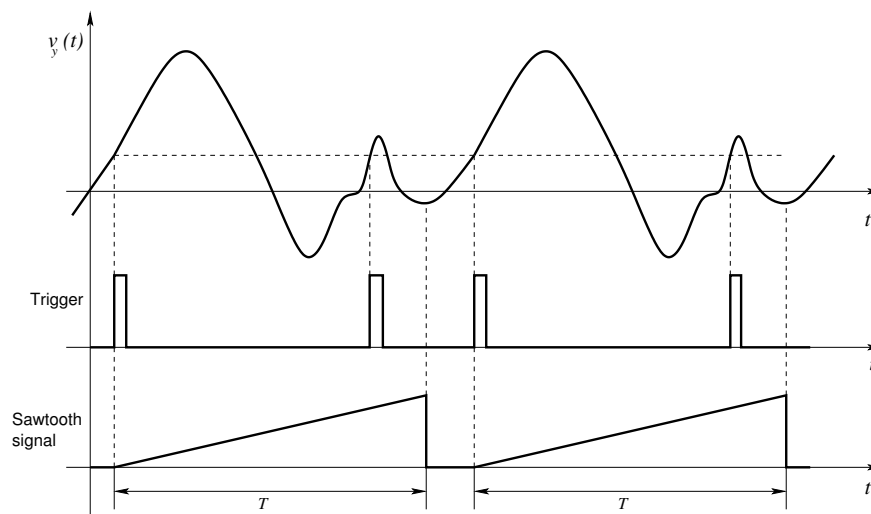


Figure A.2: Periodic Signal triggering.

### A.1.2 The Horizontal and Vertical Inputs

The vertical and horizontal plates are independently driven by a variable gain amplifier to adapt the signals  $v_x(t)$ , and  $v_y(t)$  to the screen range. A DC offset can be added to each input to position the signals on the screen. These two channels used to drive the signals to the plates signals are called horizontal and vertical inputs of the oscilloscope.

In this configuration the oscilloscope is indeed an x-y plotter.

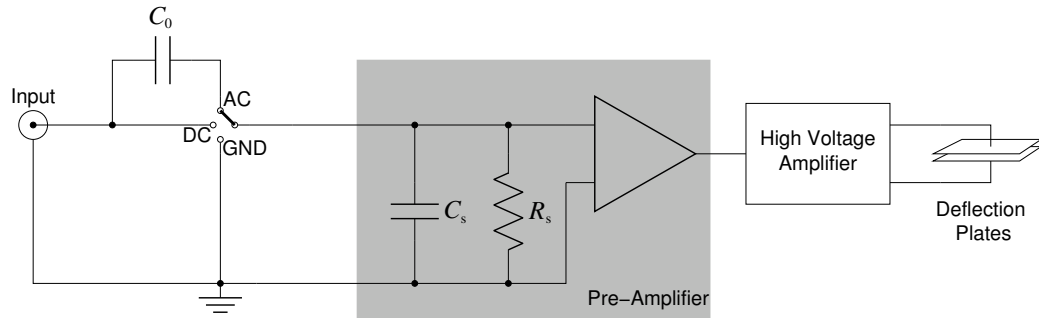


Figure A.3: Oscilloscope input impedance representation using ideal components (gray box). Input channel coupling is also shown.

### A.1.3 The Time base Generator

If we apply a sawtooth signal  $V_x(t) = \alpha t$  to the horizontal input, the horizontal screen axis will be proportional to time  $t$ . In this case a signal  $v_y(t)$  applied to the vertical input, will depict on the oscilloscope screen the signal time evolution.

The internal ramp signal is generated by the instrument, with an amplification stage that allows changes in the gain factor  $\alpha$  and in turn the interval of time shown on the screen. This amplification stage and the ramp generator are called the *time base generator*.

In this configuration, the horizontal input is used as a second independent vertical input, allowing the plot of the time evolution of two signals.

Visualization of signal time evolution is the most common use of an oscilloscope.

### A.1.4 The Trigger

To study a periodic signal  $v(t)$  with the oscilloscope, it is necessary to synchronize the horizontal ramp  $V_x = \alpha t$  with the signal to obtain a steady plot of the periodic signal. The trigger is the electronic circuit which provides this function. Let's qualitatively explain its behavior.

The trigger circuit compares  $v(t)$  with a constant value and produces a pulse every time the two values are equal and have the same slope sign.



The first pulse triggers the start of the sawtooth signal of period<sup>2</sup>  $T$ , which will linearly increase until it reaches the value  $V = \alpha T$ , and then is reset to zero. During this time, the pulses are ignored and the signal  $v(t)$  is indeed plotted for a duration time  $T$ . After this time, the next pulse that triggers the sawtooth signal will happen for the same previous value and slope sign of  $v(t)$ , and the same portion of the signal will be re-plotted on the screen.

## A.2 Oscilloscope Input Impedance

A good approximation of the input impedance of the oscilloscope is shown in the circuit of figure A.3. The different input coupling modes ( DC AC GND ) are also represented in the circuit.

The amplifying stage is modeled using an ideal amplifier (infinite input impedance) with a resistor and a capacitor in parallel to the amplifier input.

The switch allows to ground the amplifier input and indeed to vertically set the origin of the input signal (GND position), to directly couple the input signal (DC position), or to mainly remove the DC component of the input signal (AC position).

## A.3 Oscilloscope Probe

An oscilloscope probe is a device specifically designed to minimize the capacitive and resistive load added to a circuit under measurement once the instrument is connected to the circuit. The price to pay is an attenuation of the signal that reaches the oscilloscope input<sup>3</sup>.

Let's analyze the behavior of a passive probe. Figure A.4 shows the schematics of the equivalent circuit of a passive probe and of the input stage of an oscilloscope. The capacitance of the probe cable can be considered included in  $C_s$

Considering the voltage divider equation, we have

$$H(j\omega) = \frac{V_s}{V_i} = \frac{Z_s}{Z_p + Z_s}, \quad (\text{A.1})$$

<sup>2</sup>In general, the sawtooth signal period  $T$  and the period of  $v(t)$  are not equal.

<sup>3</sup>Active probes can partially avoid this problems by amplifying the signal.

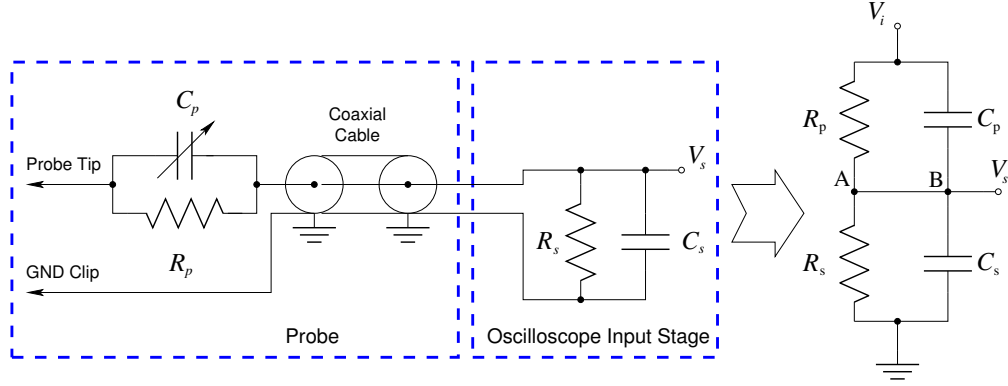


Figure A.4: Oscilloscope input stage and passive probe schematics. The equivalent circuit made of ideal components for the probe shielded cable is not shown.

where

$$\frac{1}{Z_s} = j\omega C_s + \frac{1}{R_s}, \quad \frac{1}{Z_p} = j\omega C_p + \frac{1}{R_p},$$

and then

$$Z_s = \frac{R_s}{j\omega\tau_s + 1}, \quad Z_p = \frac{R_p}{j\omega\tau_p + 1}.$$

Defining the following parameters

$$\tau_p = C_p R_p, \quad \alpha = \frac{R_s}{R_s + R_p}, \quad \beta = \frac{C_p}{C_s + C_p},$$

and after some tedious algebra, equation (A.1) becomes

$$H(j\omega) = \alpha \frac{1 + j\omega\tau_p}{1 + j\omega\frac{\alpha}{\beta}\tau_p},$$

which is the transfer function from the probe input to the oscilloscope input before the ideal amplification stage.

The DC and high frequency gain of the transfer function  $H(j\omega)$  are respectively

$$H(0) = \alpha, \quad H(\infty) = \beta.$$

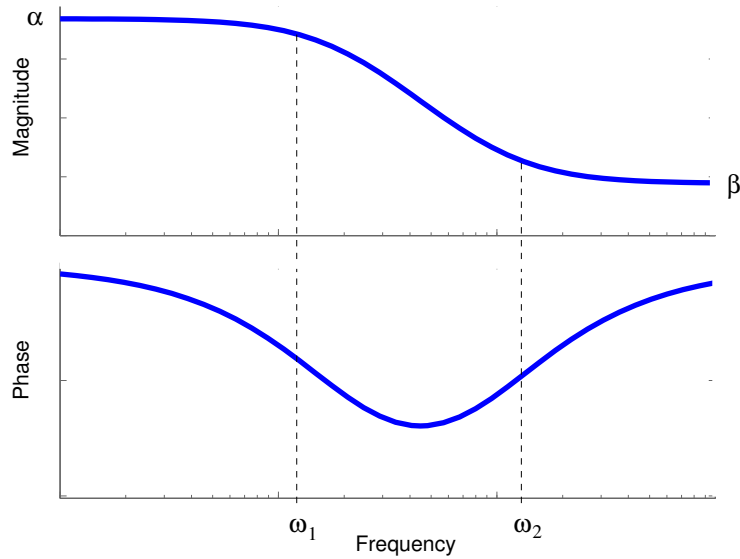


Figure A.5: Qualitative transfer function from the under compensated probe input to the oscilloscope input before the ideal amplification stage. As usual, the oscilloscope input is described having an impedance  $R_s || C_s$ .

The numerator and denominator of  $H(j\omega)$  are respectively equal to zero, (the zeros and poles of  $H$ ) when

$$\omega = \omega_z = j \frac{1}{\tau_p}, \quad \omega = \omega_p = j \frac{\beta}{\alpha} \frac{1}{\tau_p}.$$

Figure A.5 shows the qualitative behavior of  $H$  for  $\frac{\alpha}{\beta} > 1$ .

### A.3.1 Probe Frequency Compensation

By tuning the variable capacitor  $C_p$  of the probe, we can have three possible cases

$$\begin{aligned} \frac{\alpha}{\beta} < 1 &\Rightarrow \text{over-compensation} \\ \frac{\alpha}{\beta} = 1 &\Rightarrow \text{compensation} \end{aligned}$$

$$\frac{\alpha}{\beta} > 1 \Rightarrow \text{under-compensation}$$

if  $\alpha < \beta$  the transfer function attenuates more at frequencies above  $\omega_z$ , and the input signal  $V_i$  is distorted.

if  $\alpha = \beta$  the transfer function is constant and the input signal  $V_i$  will be undistorted, and attenuated by a factor  $\alpha$ .

if  $\alpha > \beta$  the transfer function attenuates more at frequencies below  $\omega_p$  and the input signal  $V_i$  is distorted.

The ideal case is indeed the compensated case, because we will have increased the input impedance by a factor  $\alpha$  without distorting the signal.

The probe compensation can be tuned using a signal, which shows a clear distortion when it is filtered. A square wave signal is very useful in this case because, it shows a different distortion if the probe is under or over compensated. Figure A.6 sketches the expected square wave distortion for the two un-compensated cases.

It is worthwhile to notice that

$$\frac{\alpha}{\beta} = 1, \quad \Rightarrow \frac{R_s}{R_p} = \frac{C_p}{C_s}.$$

This condition implies that:

- the voltage difference  $V_1$  across  $R_s$  is equal the voltage difference  $V_2$  across  $C_s$ , i.e.  $V_1 = V_2$
- the voltage difference  $V_3$  across  $R_p$  is equal the voltage difference  $V_4$  across  $C_p$ , i.e.  $V_3 = V_4$
- and indeed  $V_1 + V_2 = V_3 + V_4$ .

This means that no current is flowing through the branch AB, and we can consider just the resistive branch of the circuit to calculate  $V_s$ . Applying the voltage divider equation, we finally get

$$V_s = \frac{R_s}{R_s + R} V_i$$

The capacitance of the oscilloscope does not affect the oscilloscope input anymore, and the oscilloscope+probe input impedance  $R_i$  becomes greater, i.e.

$$R_i = R_s + R_p.$$

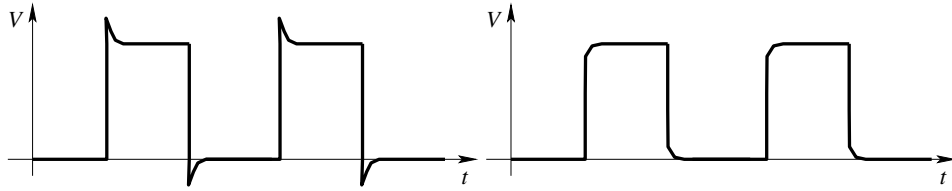


Figure A.6: Compensation of a passive probe using a square wave. Left figure shows an over compensated probe, where the low frequency content of the signal is attenuated. Right figure shows the under compensated case, where the high frequency content is attenuated.

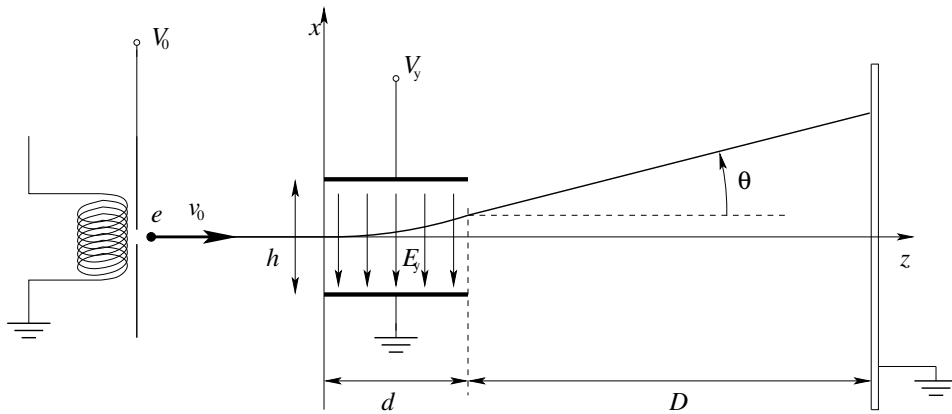


Figure A.7: CRT tube schematics. The electron enters into the electric field and makes a parabolic trajectory. After passing the electric field region it will have a vertical offset and deflection angle  $\theta$ .

## A.4 Beam Trajectory

Let's consider the electron motion through one pair of plates.

The electron terminal velocity  $v_0$  coming out from the gun can be easily calculated considering that its initial potential energy is entirely converted into kinetic energy, i.e

$$\frac{1}{2}\mu v_0^2 = eV_0, \quad \Rightarrow \quad v_0 = \sqrt{2\frac{eV_0}{\mu}},$$

where  $\mu$  is the electron mass,  $e$  the electron charge, and  $V_0$  the voltage applied to the last anode.

If we apply a voltage  $V_y$  to the plates whose distance is  $h$ , the electrons will feel a force  $F_y = eE_y$  due to an electric field

$$|E_y| = \frac{V_y}{h}.$$

The equation of dynamics of the electron inside the plates is

$$\begin{aligned} \mu \ddot{z} &= 0, \quad \Rightarrow \quad \dot{z} = v_0, \\ \mu \ddot{y} &= e|E_y|. \end{aligned}$$

Supposing that  $V_y$  is constant, the solution of the equation of motion will be

$$\begin{aligned} z(t) &= \sqrt{2\frac{eV_0}{\mu}}t, \\ y(t) &= \frac{1}{2}\frac{eV_y}{\mu h}t^2. \end{aligned}$$

Removing the dependency on the time  $t$ , we will obtain the electron beam trajectory, i.e.

$$y = \frac{1}{4h}\frac{V_y}{V_0}z^2,$$

which is a parabolic trajectory.

Considering that the electron is transversely accelerated until  $z = d$ , the total angular deflection  $\theta$  will be

$$\tan \theta = \left( \frac{\partial y}{\partial z} \right)_{z=d} = \frac{1}{2} \frac{d}{h} \frac{V_y}{V_0}.$$

and displacement  $Y$  on the screen is

$$Y(V_y) = y(z = d) + \tan \theta D,$$

i.e.,

$$Y(V_y) = \frac{1}{2} \frac{d}{h} \frac{1}{V_0} \left( \frac{d}{2} + D \right) V_y.$$

$Y$  is indeed proportional to the voltage applied to the plates through a rather complicated proportional factor.

The geometrical and electrical parameters of this proportional factor play a fundamental role in the resolution of the instrument. In fact, the smaller the distance  $h$  between the plates, or the smaller the gun voltage drop  $V_0$ , the larger is the displacement  $Y$ . Moreover,  $Y$  increases quadratically with the electron beam distance  $d$ .

### A.4.1 CRT Frequency Limit

The electron transit time through the plates determine the maximum frequency that a CRT can plot. In fact, if the transit time  $\tau$  is much smaller than the period  $T$  of the wave form  $V(t)$ , we have

$$V(t) \simeq \text{constant}, \quad \text{if } \tau \ll T,$$

and the signal is not distorted.

The transit time is

$$\tau = \frac{d}{v_0} = d \sqrt{\frac{\mu}{2eV_0}}.$$

Supposing that

$$\begin{cases} V_0 = 1\text{kV} \\ d = 20\text{mm} \\ \mu c^2 \simeq 0.5\text{MeV} \\ e = 1\text{eV} \end{cases} \Rightarrow \tau \simeq 1\text{ns}$$